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# Robust radar waveform algorithm based on beam steering vector estimation

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**Abstract.** In view of the problems of traditional adaptive beam-forming algorithm in the practical application, this paper, under the premise of acquisition of expected target angular domain based on the sequence of two quadratic programming method, puts forward the robust adaptive beam-forming based on beam-space steering vector estimation (BA-RAB). First of all, make use of the complement structure in the expected beam domain angle area to form the weights, and then the weights are reversely transformed to matrix domain. As a result, it can obtain adaptive robust with steady performance to form weights on the two occasions of steering vector mismatch and training samples contaminated. Moreover, the effectiveness of the proposed method is verified by computer simulation.

**Key words.** Radar waveform, algorithm, adaptive beam.

## 1. Introduction

Although all solid state transmitter and digital receiver greatly improve the stability of radar, the system error is greatly reduced. The significant improvement of the technological level of the antenna makes the channel error further reduced, but it cannot completely eliminate the array channel amplitude and phase error. At the same time, the estimation error and steering vector error of signal co-variance matrix caused by finite samples or small samples will lead to the decrease of the adaptive beam-forming performance.

Reasonable use of angle region information of the target signal can eliminate the impact of training samples being polluted. This paper uses the complement structure beam domain conversion matrix of the desired signal beam domain angle mismatch region, to transform the array domain training data to the beam field, which eliminates the desired signal component in the training samples, and gives the

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beam-space steering vector estimation method, using semi definite relaxation planning solution with constant constraint quadratically constrained quadratic programs (QCQP) problem [1].

## 2. Literature review

The generalized radar waveform includes transmitting wave and receiving wave. Received waveform is the waveform matched with the radar receiver filter. When the radar is carry out mismatch processing with the received signal, the received waveform is different from the transmitted waveform. Radar waveform is of variety, and according to the fuzzy function form, it can be divided into three types: 1. the signal with blade fuzzy function, including constant frequency pulse signal with positive blade fuzzy function and linear frequency modulation pulse signal with inclined blade fuzzy function; 2. the pseudo-random signal with encoding thumbtack fuzzy function; 3. the coherent pulse train signal with the nail bed type fuzzy function. In addition to the constant frequency pulse signal, for other types of signal, the multiple of time bandwidth and bandwidth is greater than 1, referred to as the large time bandwidth product signal. The signal with inclined blade type and thumbtack fuzzy function is the pulse width encoding signal. Due to the introduction of nonlinear phase modulation in the pulse, wide pulse with narrow pulse bandwidth, through the matched filter or related integral processing, can be compressed into a narrow pulse to output, called the pulse compression signal. This signal not only has excellent detection performance and speed performance, but also has high range resolution and ranging performance of narrow pulse waveform. Pulse compression waveform can be divided into two types of frequency modulated pulse compression signal and phase coded pulse compression signal according to the nonlinear phase modulation rule. The former also has the difference in linear frequency modulation and the nonlinear frequency modulation; the latter has the difference in two-phase code, the multiple-phase code and the complementary code.

The traditional adaptive beam-forming algorithm is applied to practical problems, there are two aspects of the problem. On the one hand, the traditional adaptive beam-forming algorithm assume that there is no desired signal in the training data, but in practice, if the desired signal exists in the training data, it will significantly reduce the convergence rate of the adaptive beam-forming algorithm. If the number of samples is very small, it will lead to a serious decline in performance. On the other hand, the traditional adaptive beam-forming algorithm, under non-ideal conditions, such as environmental pollution and the antenna array pollution, it will be affected by the pollution, which will lead to some errors between the steering vector assumed and the actual signal steering vector, and the traditional adaptive beam-forming algorithm is very sensitive to steering vector error. Very sensitive, then the algorithm performance also very serious declines. The researchers showed that the traditional adaptive beam-forming method is sensitive to the steering vector mismatch and array error [2]. To get high performance, it is necessary to know the co-variance matrix of the desired signal steering vector and the interference plus noise. In the actual scene, there is always mismatch reduction between the desired signal steering vector assumed and the actual steering vector [4]. In addition, in the passive no-source positioning, mobile communication and so on actual application cases, the training sample data will inevitably contain the desired signal component, oriented vector mismatch and the training samples containing the desired signal component, which will cause the decline of the performance of adaptive beam-forming [5]. The research of adaptive beam forming algorithm with steady performance is the problem needed to be solved in engineering application.

There exist many methods used to improve the robustness of adaptive beam-forming algorithm. Literature [3] makes use of diagonally Loaded SMI, LSMI sample matrix inversion method to improve the direct sample matrix inversion (SMI) robust beam-forming, and LSMI, by controlling the diagonal loading to increase artificial white noise variance, alleviates the self-destructive problem of useful signal, suitable for mismatch cases in different types, but the method has an obvious deficiency - it is difficult to determine the amount of diagonal loading. Literature [2], in allusion to the steering vector mismatch, proposed robust beam-forming with mode constraint algorithm, and acquired the robust adaptive algorithm relevant with the uncertainty set, while the method needs appropriate parameters selection to ensure its robustness. Literature [5] studied the robust adaptive beam-forming algorithm under the worst performance (Worst case based RAB, W C-RAB). On the assumption that the mismatch scale limit is known, the approximate analytic formula of the optimal loading amount is obtained, while the limit of mismatch amount in practice is not known. The literature [6] and literature [9] improve the literature [5], and under the assumption that the spatial range of the target signal uses the low resolution method, the iterative sequence of quadratic programming (Sequential Quadratic Programming, SQP) method is used to estimate the actual steering vector. However, since that the impact of the desired signal component in the training samples is not considered, when the signal-to-noise ratio increases, the increase of output SINR will occur bottlenecks.

## 3. Method

The proposed algorithm is based on the premise of target angular domain known by low resolution method. First of all, use the complement structure beam-space conversion matrix of the desired signal angle area, and then, in the beam domain, deduct the estimation algorithm of the desired steering vector, and calculate beam-space beam-forming weights. At last, the beam-space beam-forming weights are transformed to the element space, which can obtain adaptive beam-forming weight with robust performance on the two conditions for steering vector mismatch and training samples contaminated.

### *3.1. Robust radar waveform formation algorithm based on beam steering vector estimation*

The algorithm consists of six steps:

1. According to the low resolution method, divide the target airspace range $\Theta$

and the interference target airspace $\bar{\Theta}$.

2. Calculate $A = \int_{\bar{\Theta}} a(\theta)a^{\mathrm{H}}(\theta)\,\mathrm{d}\theta$, where $a(\theta)$ is the expected signal steering vector and $\theta$ represents the signal location. The number of $A$'s main feature vectors $M'$ is derived from the minimum description length criterion (MDL) method. Construct the beam transform matrix $B = [u_1,\, u_2,\, \cdots\, u_{M'}]_{M\times M'}$, in which $\{u_i\}_{i=1}^{M'}$ is the $M'$th main feature vector of $A$.

3. Calculate the co-variance matrix $R_b = B^{\mathrm{H}} R_\chi B$ of the beam field, and make use of CVX to solve (1) that does not contain rank $(Z_{\mathrm{b}}) = 1$, and obtain the optimal solution in the form

$$\begin{cases} \min_{Z_{\mathrm{b}}} \mathrm{Tr}(\hat{R}_{\mathrm{b}}^{-1} Z_{\mathrm{b}})\,, \\ \mathrm{s.t.Tr}(BB^{\mathrm{H}})^{-1} B Z_{\mathrm{b}} B^{\mathrm{H}}((BB^{\mathrm{H}})^{-1})^{\mathrm{H}} = M\,, \\ \mathrm{Tr}(Z_{\mathrm{b}}) \le \delta_0\,, \\ Z_{\mathrm{b}} \succeq 0\,, \\ \mathrm{rank}(Z_{\mathrm{b}}) = 1\,, \end{cases} \tag{1}$$

$\delta_0$ being the unknown error between the expected and true steering vectors and

4. Randomly generate $L$ vectors $\xi_l \sim N(0, \hat{\mathrm{Z}}_{\mathrm{b}})$ following zero mean Gauss distribution, $l = 1, \ldots, L$. They are normalized, and then $\tilde{a}_l = \xi_l / \|\xi_l\|$ is obtained. The chosen optimal beam steering vector $\hat{a}_b = \tilde{a}_l$ meets the condition $l^* = \arg \min\limits_{l=1,\ldots,L} \tilde{a}_l^{\mathrm{H}} R_{\mathrm{b}}^{1} \tilde{a}_l$ ($\hat{a}_{\mathrm{b}}$ denoting the optimal signal steering vector and $\tilde{a}_b$ denoting the true signal steering vector).

5. After obtaining the optimal estimate beam steering vector $\hat{a}_{\mathrm{b}}$, make use of (2) to solve the beam domain Capon weight vector $W_{\mathrm{b\_opt}}$;

$$w_{\mathrm{b}} = \frac{\hat{R}_{\mathrm{b}}^{-1} \tilde{a}_{\mathrm{b}}(\theta_{\mathrm{s}})}{\tilde{a}_{\mathrm{b}}^{\mathrm{H}}(\theta_{\mathrm{s}}) \hat{R}_{\mathrm{b}}^{-1} a_{\mathrm{b}}^{\mathrm{H}}(\theta_{\mathrm{s}})}\,. \tag{2}$$

6. Eventually, make use of the conversion relation $W_{\mathrm{opt}} = (B\,B^{\mathrm{H}})^1 B W_{\mathrm{b\_opt}}$ between the beam domain Capon vector and right array domain Capon weight vector to solve the domain Capon adaptive weight vector $W_{\mathrm{opt}}$.

### *3.2. Simulation experiment design*

The effectiveness of the proposed method is verified by computer simulation experiments. Assume that the receiving array is a uniform linear array, the number of array elements is $M = 20$, and the array element spacing is half of the wavelength. It is assumed that the two interference sources are incident at 30 degrees and 50 degrees respectively, the dry noise ratio is 30 dB, the actual incident angle of the desired signal is $\theta_{\mathrm{P}} = 5°$, and the space noise is Gauss white noise with zero mean value. All the simulation results are obtained by 200 Monte-Carlo experimental statistics, and the training snapshot data always contains the desired signal component. The proposed algorithm (BS-RAB) is compared with SMI, LSMI, WC-RAB and SQP-RAB. For BS-RAB and SQP-RAB, the angle domain interval of the desired signal is assumed as $\Theta = [\theta_{\mathrm{P}} + 5^{\circ}\ \text{ to }\ \theta_{\mathrm{P}} - 5^{\circ}]$, the number of main feature value is $M' = 8$, and the proposed algorithm BS-RAB theory is $N_{3\mathrm{dB}} = 13$. Slack variable of SQP-RAB is $\delta_0 = 0.1$. The upper limit of steering vector mismatch amount in W C-RAB algorithm, is set to $\varepsilon = 0.3M$, and the diagonal loading factor of LSMI algorithm is set to two times of the noise power.

Table Simulation experiments assumptions

| Simulation experiments | |
|---|---|
| Experiment 1 | In the case of existing direction mismatch, compare the performance of BS-RAB, SMI, LSMI, WC-RAB and SQP-RAB algprithms. The assumed steering vector is calculated by $\theta_{\mathrm{p}}+\theta_{\mathrm{e}}$, $\theta_{\mathrm{e}}$ being observing direction mismatch error, and in the experiment, $\theta_{\mathrm{e}}$ is set to evenly distributed in the interval $[-5^{\circ}, 5^{\circ}]$. |
| Experiment 2 | In the construction of beam domain conversion matrix, make use of the desired target angle domain O and the array element position information without interference. The desired target angle domain is the mismatch region of the target direction, and the selection criteria is only containing the useful signal, but not containing any interference direction. It is assumed that the element position interference caused by antenna position error is evenly distributed in $[-k\lambda, k\lambda]$, where $\lambda$ is the wavelength. |
| Experiment 2 | Considering the direction mismatch and element position interference, compare the performance of BS-RAB, SMI, LSMI, WC-RAB and SQP-RAB algorithms. Assume that the error between the element and ideal position is evenly distributed in the interval of [-0.15$\lambda$, 0.15$\lambda$], where $\lambda$ is the wavelength. Other simulation conditions are the same as in experiment 1 settings. |

## 4. Results

Experiment 1: set the single element SNR is SNR=l0 dB, compares the change relation of 5 algorithms output SINR with the training snapshots, as shown in Fig. 1. As can be seen from the figure, WC-RAB and SQP-RAB, on the condition that the number of snapshots is less than 30, have a similar output SINR; while on the condition that the number of snapshots is larger than 30, the output of SQP-RAB SINR is better than that of WC-RAB, and the algorithm proposed BS-RAB output SINR has basically reached the convergence value in the output snapshots of 20. In the case of small samples, this method has better performance, while the overall output SINR is much higher than the other 4 algorithms.

The number of snapshots is set as 100, then the variation relation diagram of 5 algorithms output SINR with the input SNR is shown in Fig. 2. We can see that, when the input SNR approximation is less than 20 dB, the output of SQP-RABSNR

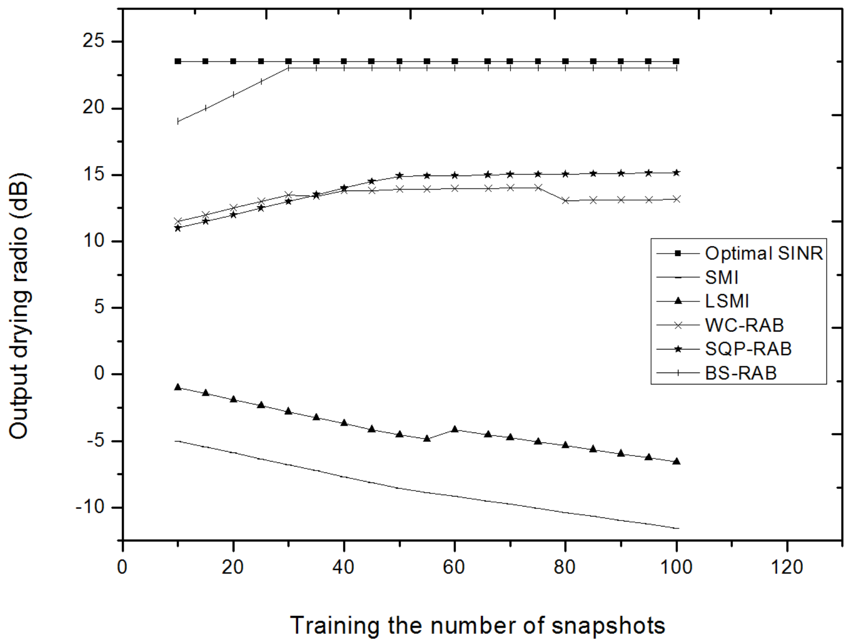


Fig. 1. The variation curves of OSINR with the number of snapshots on the occasion of direction mismatch

SINR is higher than that of WC-RAB; when the input SNR approximation is larger than 20 dB, the output of WC-RAB SINR is higher than that of SQP-RAB. But there is bottleneck in the variation of SINR with the increase of SNR, while the output of BS-RAB SINR has the performance of close to the optimal SINR. It can be seen that BS-RAB has a strong robust performance to signal direction mismatch error, and the output of SINR is superior to other algorithms.

## 5. Discussion

The method proposed in this paper does not require the upper limit constrain of the steering vector mismatch error, which avoids the modeling of the upper limit of the steering vector mismatch. Compared with the method of literature [6], although all belong to the class of robust adaptive steering vector estimation algorithm, because the proposed method reasonably uses the spatial information of target signal and interference, the proposed method can eliminate the influence of the desired signal contained in the training sample. In addition, since that the steering vector of non-uniform linear array is more general compared with uniform linear array, and there is no limit in the construction of beam-space transformation matrix and the constrain condition of estimation beam-space steering vector in the array type, the proposed method is also applicable to non-uniform linear array.

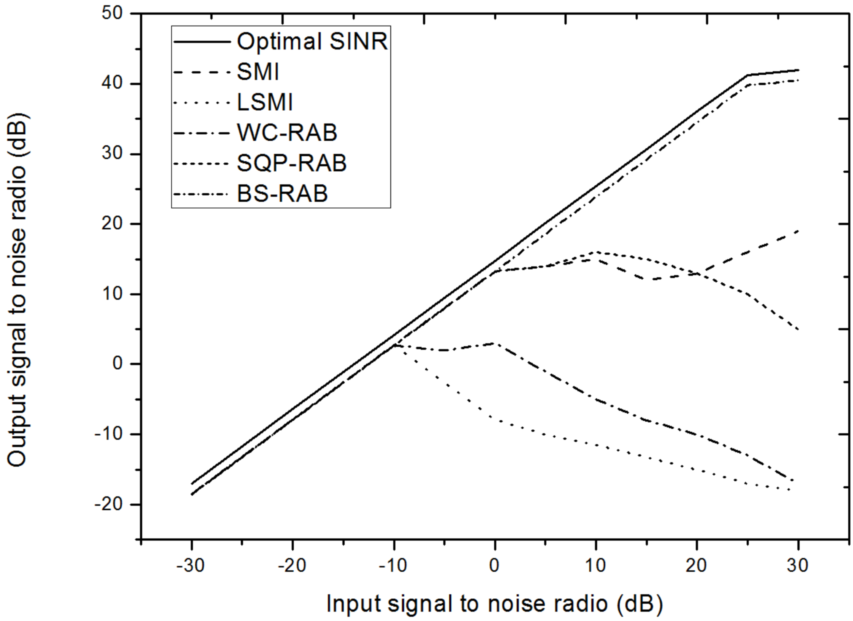


Fig. 2. The performance comparison on the occasion of direction mismatch

Experiment 2: Figure 3 shows the variation curves of constrained region $\hat{\Theta}_k$ deviation degree from $\hat{\Theta}$ with $k$. It is found that the $\hat{\Theta}_k$ deviation is less than 4% in the presence of the array element, and the influence of the element disturbance on the deviation is not great. Although there is a difference between the use of the steering vector without array position disturbance and the steering vector calculation matrix $A$ in the presence of array element position disturbance, as long as the angle domain $\hat{\Theta}_k$ contains only the desired signal but does not contain any interference signal, the constrain $\|\tilde{a}_b\| \leq \max \|\tilde{a}_b(\theta)\|$, $\theta \in \Theta$ is established. As a result, we can calculate the beam field transformation matrix by the assumption that the array elements are not perturbed.

The simulation experiment 3 showed that the proposed method is suitable for small sample snapshots, and compared with other algorithms, it has more performance advantages.

## 6. Conclusion

This paper studies the robust adaptive beam-forming method, and for steering vector mismatch and sample co-variance matrix estimation error resulting in adaptive beam-forming performance degradation problems, puts forward a kind of robust adaptive beam-forming method (BA-RAB) based on beam-space steering vector estimation on the condition of training samples contaminated. This method uses the

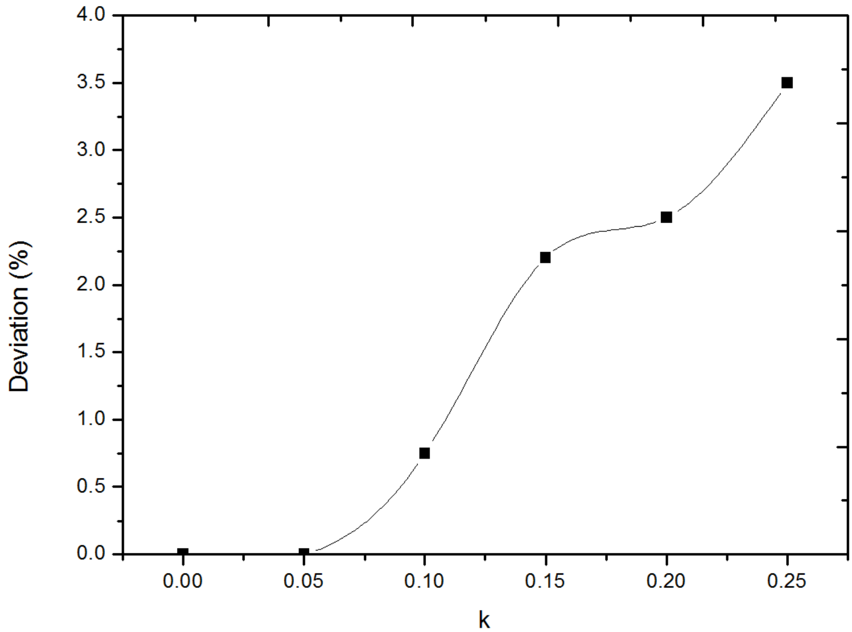


Fig. 3. The variation curve of $\hat{\Theta}_k$ deviation with element disturbance degree

complement structure of the desired signal beam domain angle mismatch area of the conversion matrix, the array domain training data is transformed to the beam field, to eliminate the desired signal component in the training samples, and gives the beam-space steering vector estimation method, using semi definite relaxation planning solving QCQP problem with constant modulus constraint. The proposed method, in the case of training sample contaminated and steering vector mismatched, can effectively improve the robust performance of adaptive beam-forming output SINR, so the output SINR achieves the approximate optimal SINR, and it is suitable for small sample snapshot. It overcomes the shortcomings of the existing algorithms useful signal elimination when the desired signal component exists in the sample covariance matrix. And then the method to estimate the beam-space steering vector is derived, and transformed into quadratic programming problems with two non-convex constant modulus constraint, then the semi definite relaxation planning is used to estimate the actual beam-space steering vector. Finally, the effectiveness of the proposed method is verified by computer simulation.

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